RELATIVITY & HIGH ENERGY PHYSICS

PARTICLE OF MASS m.

MOMENTUM:

Classical Physics $\vec{p}_{cl} = m\vec{v}$

Relativity : $$\boxed{\vec{p} = \frac{m\vec{v}}{\sqrt{1 - v^2/c^2}}} \qquad (1)$$

For $v \ll c$ $\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}} \cong 1 + \frac{1}{2}\frac{v^2}{c^2} + \ldots \approx 1.$

ENERGY:

$$\boxed{E = \frac{mc^2}{\sqrt{1 - v^2/c^2}}} \qquad (2)$$

When $v \ll c$ $$E = mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \ldots\right)$$

$$\approx \underbrace{mc^2}_{\text{rest energy}} + \underbrace{\frac{1}{2}mv^2}_{E_{cl} \to \text{Kinetic Energy}} + \ldots$$

MASS m ALWAYS REFERS to REST MASS.

~~PARTICLE OF MASS ZERO~~

PROTON: $mc^2 = 938$ MeV, PION : $mc^2 = 138$ MeV

ELECTRON: $mc^2 = 0.511$ MeV MUON: 105 MeV

~~MUON~~ :

PHOTON: $mc^2 = 0$ W, Z : $mc^2 = 80.33$ GeV, 91.87 GeV

Identity:

$$E = \sqrt{p^2c^2 + m^2c^4}$$

Pf.

$$E^2 = p^2c^2 + m^2c^4$$

$$\left(\frac{mc^2}{\sqrt{1-v^2/c^2}}\right)^2 - \left(\frac{mv}{\sqrt{1-v^2/c^2}}\right)^2 c^2 \cancel{+m^2c^4}$$

$$= \frac{1}{1-v^2/c^2} m^2c^4\left[1 - \frac{v^2}{c^2}\right] = m^2c^4 \quad \blacksquare$$

ZERO MASS PARTICLES

$m = 0 \Rightarrow E = pc$

- Photons
- Gluons

Approximately zero mass

- Quarks — u, d, 5 MeV
- Electrons
- Neutrinos

Zero mass particles always travel at $v = c$. (in all frames)

(1) and (2) do not define $\vec{p}$ and E

Plank formula: $E = h\nu$; $p = \frac{E}{c} = \frac{h\nu}{c}$

E.

2eV photon is red

3eV photon is purple

3-VECTORS

$\vec{A} = (A_x, A_y, A_z)$

INNER PRODUCT

$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$

Length $\sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{\vec{A}\cdot\vec{A}}$

ROTATIONS:

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

ROTATIONS DO NOT CHANGE LENGTH OF A

$\vec{A}\cdot\vec{B}$ doesn't vary with rotations

4-VECTORS

$$\mathcal{A} = (\underset{\substack{\uparrow \\ \text{time component}}}{A_0}, \underbrace{A_1, A_2, A_3}_{\text{space components}})$$

example $\quad x = (ct, x, y, z)$

$\quad p = (\frac{E}{c}, p_x, p_y, p_z)$

~~LORENTZ TRANSFORMATIONS~~

INNER PRODUCT

$$\mathcal{A}\cdot\mathcal{B} \equiv A_0 B_0 - A_1 B_1 - A_2 B_2 - A_3 B_3$$

LORENTZ TRANSFORMATIONS DO NOT CHANGE

$\mathcal{A}\cdot\mathcal{B}$

"FAMOUS"
INNER PRODUCTS

$$x \cdot x = (ct)^2 - x^2 - y^2 - z^2 \equiv \tau^2$$ Proper Time

$$\tau = \sqrt{(ct)^2 - (x^2 + y^2 + z^2)}$$
$$= \sqrt{x \cdot x}$$

TIME ON CLOCK TRAVELING ALONG WITH MOVING PARTICLE

$$p \cdot p = \frac{E^2}{c^2} - (\vec{p})^2 = m^2 c^2$$ MASS

$$m = \frac{1}{c} \sqrt{p \cdot p}$$

It is very useful that the inner product of a 4-VECTOR is frame independent

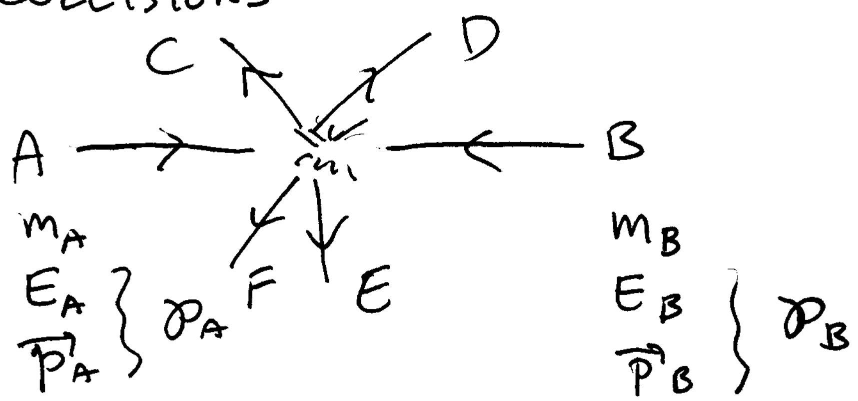
We can compute it in any frame

Often write $\quad p = \left(\frac{E}{c}, \vec{P} \right)$

REST FRAME $\quad p = \left(\frac{mc^2}{c}, 0 \right)$

ENERGY & MOMENTUM CONSERVATION

COLLISIONS



Classical Collisions: $(H_2S + O \rightarrow H_2O + S)$
$(2H + O \rightarrow H_2O)$

1. Mass Conserved $\quad m_A + m_B = m_C + m_D$
2. Momentum Conserved $\quad \vec{p}_A + \vec{p}_B = \vec{p}_C + \vec{p}_D$
3. Kinetic energy may or may not be conserved.

Type of Collisions $\Big\langle$ elastic (K.E. conserved)
inelastic (K.E. not conserved)

a). STICKY: K.E. decreases $\quad T_A + T_B > T_C + T_D$
(Endothermic)

b) Explosive: K.E. increases $\quad T_A + T_B < T_C + T_D$
(Exothermic)

c) Elastic: K.E. conserved $\quad T_A + T_B = T_C + T_D$

a) and b) $\rightarrow$ BINDING ENERGY absorbed or released.

Relativistic Collisions

1. Energy Conserved $E_A + E_B = E_C + E_D$

2. MOMENTUM CONSERVED $\vec{p}_A + \vec{p}_B = \vec{p}_C + \vec{p}_D$

3. K.E. maybe conserved.

TYPES:

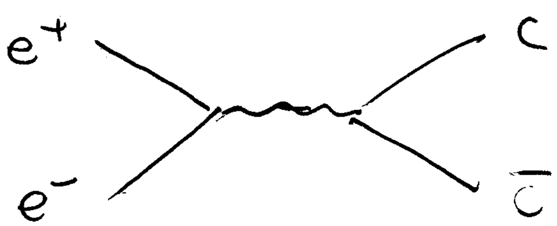
a) Sticky : rest mass increases

b) Explosive : rest mass decreases

c) Elastic : rest mass stays same

& K.E. conserved.

Sticky:



$m_c c^2 = 1.8$ GeV

This is how the

$$2(m_e c^2) = 1 \text{ MeV} < 2(1.8 \text{GeV}) = 3.6 \times 10^3 \text{ MeV}$$

Explosive: J/ψ (3.1 GeV) $\rightarrow e^+ e^-$

A PION AT REST DECAYS INTO A MUON AND A NEUTRINO. ASSUME $m_\nu = 0$

a) ~~E~~ FIND THE ENERGY OF THE OUTGOING MUON

m_π, m_μ are non zero.

$$E_{before} = m_\pi c^2 \qquad \vec{p}_{before} = 0$$

$$E_{after} = E_\mu + E_\nu \qquad \vec{p}_{after} = \vec{p}_\mu + \vec{p}_\nu$$

CONSERVATION OF MOMENTUM $\Rightarrow$ $\vec{p}_\mu = -\vec{p}_\nu$

CONS. OF ENERGY $\Rightarrow$ $E_\mu + E_\nu = m_\pi c^2$

$m_\nu = 0 \Rightarrow \quad E_\nu = p_\nu c$

$$\cancel{p_\mu} \quad E_\mu^2 = p_\mu^2 c^2 + m_\mu^2 c^4$$

$$\hookrightarrow \quad p_\mu = \frac{\sqrt{E_\mu^2 - m_\mu^2 c^4}}{c}$$

$$E_\mu + E_\nu = m_\pi c^2$$

$$E_\mu - m_\pi c^2 = E_\nu = p_\nu c = p_\mu c$$

$$(E_\mu - m_\pi c^2)^2 = p_\mu^2 c^2 = E_\mu^2 - m_\mu^2 c^4$$

$$\cancel{E_\mu^2} - 2 m_\pi c^2 E_\mu + m_\pi^2 c^4 = \cancel{E_\mu^2} - m_\mu^2 c^4$$

$$\boxed{E_\mu = \frac{1}{2 m_\pi c^2} \cancel{\sqrt{(m_\pi^2 + m_\mu^2) c^4}} = \frac{1}{2 m_\pi} (m_\pi^2 + m_\mu^2) c^2}$$

ALTERNATE way to do this problem using 4-VECTORS

3a)

PION AT REST DECAYS INTO A MUON AND A NEUTRINO

a) Find E_μ of the outgoing muon in terms of the masses m_π and m_μ. Assume $m_\nu = 0$

b) Find the velocity v_μ of the muon in terms of m_π and m_ν.

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a) 4-VECTOR $\quad p_\pi^\alpha = p_\mu^\alpha + p_\nu^\alpha$

$$\cancel{(p_\pi^\alpha)^2} \qquad (p_\pi)^\alpha (p_\pi)_\alpha = m^2 c^2$$

$$= \left(\frac{E_\mu + E_\nu}{c}\right)^2 - (\vec{p}_\mu + \vec{p}_\nu)^2$$

CM frame $\vec{p}_\mu = -\vec{p}_\nu$

$$m_\pi^2 c^4 = (E_\mu + E_\nu)^2 \qquad \boxed{m_\pi c^2 = E_\mu + E_\nu \quad \text{— (1)}}$$

$E_\nu = p_\nu c$ < massless

$= p_\mu c$ < $\vec{p}_\mu = -\vec{p}_\nu$

$= \sqrt{E_\mu^2 - m_\mu^2 c^4}$ < $E_\mu^2 = p_\mu^2 c^2 + m_\mu^2 c^4$

$$\cancel{m_\pi^2 c^4} = (m_\pi c^2 - E_\mu)^2 = E_\nu^2 = \cancel{E_\mu^2} - m_\mu^2 c^4$$

$$= (m_\pi c^2)^2 + \cancel{E_\mu^2} - 2 m_\pi c^2 E_\mu$$

$$E_\mu = \frac{1}{2 m_\pi c^2} (m_\pi^2 + m_\mu^2) c^4$$

b) Find velocity of the outgoing muon (4)

$$v = \frac{p_\mu c^2}{E_\mu} \qquad \frac{v}{c} = \frac{p_\mu c}{E_\mu} = \frac{1}{E_\mu}\sqrt{E_\mu^2 - m_\mu^2 c^4}$$

$$E_\mu^2 - m_\mu^2 c^4 = \frac{(m_\pi^2 + m_\mu^2)^2 c^4}{4 m_\pi^2} - m_\mu^2 c^4$$

$$= \frac{c^4}{4m_\pi^2}\left[(m_\pi^2 + m_\mu^2)^2 - 4 m_\pi^2 m_\mu^2 \right]$$

$$= \frac{c^4}{4m_\pi^2}\left[m_\pi^4 + m_\mu^4 + 2 m_\mu^2 m_\pi^2 - 4 m_\mu^2 m_\pi^2 \right]$$

$$= \frac{c^4}{4m_\pi^2} (m_\pi^2 - m_\mu^2)^2$$

$$\frac{v}{c} = \frac{2m_\pi}{(m_\pi^2 + m_\mu^2) c^2} \frac{c^2}{2m_\pi} (m_\pi^2 - m_\mu^2) = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2}$$

$$\boxed{v = \left(\frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} \right) c}$$

PHOTON INCIDENT ON STATIONARY PROTON

$$\gamma + p \rightarrow n + \pi^+$$

this reaction will produce charged pions.
FIND LOWEST E_γ at which π^+ are produced

★ PROTON AT REST IN LAB FRAME

$$\left.\begin{aligned} p_p^\mu &= (m_p c, 0) \\ p_\gamma^\mu &= \left(\frac{E_\gamma}{c}, \vec{p}_\gamma\right) \end{aligned}\right] \quad \text{Lab frame}$$

★ LOWEST ENERGY: neither n nor π^+ have any kinetic energy IN CM frame

$$P^\mu_{TOT,CM} = (m_n c + m_\pi c, 0)$$

$P_i^\mu = P_f^\mu$ CONS. OF ENERGY/MOMENTUM

$$\underbrace{P_i^\mu P_{i\mu}}_{\text{Calculate in lab frame}} = \underbrace{P_f^\mu P_{f\mu}}_{\text{Calculate in CM frame}}$$

$$P_i^\mu P_{i\mu}\Big|_{LAB} = \left(m_p c + \frac{E_\gamma}{c}\right)^2 - p_\gamma^2 = \left(m_p c + \frac{E_\gamma}{c}\right)^2 - \left(\frac{E_\gamma}{c}\right)^2$$

$$P_f^\mu P_{f\mu}\Big|_{CM} = (m_n + m_\pi)^2 c^2$$

Setting them equal

$$m_p^2 c^2 + 2 m_p E_\gamma = (m_n + m_\pi)^2 c^2$$

$$\boxed{E_\gamma = \frac{(m_n + m_\pi)^2 c^2 - m_p^2 c^2}{2 m_p}}$$

~~NEUTRON β DECAY~~

~~$\cancel{\text{If the}}\; n \to p + e^- + \bar{\nu}_e$~~

~~Neutron initially at rest.~~

~~Find maximum velocity for the electron~~