Momentum-Multiplicity Correlations in Relativistic Heavy Ion Collisions

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1 Introduction

My REU project focused on high-energy nuclear collisions. I collaborated with past REU student Michael Catanzaro and faculty mentor Sean Gavin to study momentum-multiplicity correlations. This work extends work performed last year by Catanzaro in two ways (see Catanzaro’s REU report from summer 2009 on this web site). First, I studied pion and kaon distributions to see how correlations appear in each particle species. Second, I used the Monte Carlo event generator HIJING [1] to see how the phenomena of jet quenching can affect these correlations.

Momentum-multiplicity correlations can be used to study the role of mini-jets and radial flow in nuclear collisions. These very different phenomena both enhance the transverse momenta of particles produced in the nuclear collisions. Measured nuclear effects in elliptic flow and particle correlations have been attributed to both effects; see Catanzaro’s report for references. There is little doubt that both effects can contribute to these measurements at some level, so it is important to find an experimental way to distinguish these effects. In their project last year, Catanzaro and Gavin proposed an observable $D$, explained below, that can distinguish the mini-jet and flow contributions. I ran simulations using the Monte Carlo event generator HIJING [1] to test a relationship which scales the results for $D$ in p-p collisions to that obtained in nuclear collisions.

Here I begin by explaining our motivation for calculating $D$ specifically by explaining the difference between jets (and mini-jets) and flow. In sec 2, I introduce the quantity $D$. I then briefly explain the independent source model and why this lends us to use HIJING for the simulations before looking at some preliminary results of the calculations.

1.1 Jets

Jets occur in both particle and nuclear physics experiments. Though rare, this high-energy phenomenon is caused when there is a head-on collision between two partons (quarks or gluons). As the partons hard-scatter, they fragment to form a narrow cone of several particles with high transverse momenta.

Mini-jets are similar processes which occur at much lower energies than jets. For the purposes of this paper, mini-jets can be understood in the same manner as jets.

1.2 Radial Flow

As the nuclei in heavy ion experiments crash through each other, flux tubes form along the beam axis. These flux tubes stretch and fragment to form particles with low transverse momenta. This process continues to occur and more and more particles are created near the beam axis. These particles then act as a pressure on the peripheral particles which are pushed radially outward from the beam axis giving them greater and greater transverse momenta.

2 Calculating $D$

Correlation measurements are of much interest to those studying nuclear collisions. Measurements of two particle correlations provided the first evidence of jet quenching. I now closely follow work done by Catanzaro and begin by mentioning that new studies focus on the two-body correlation functions

$$r(p_1, p_2) = \rho_2(p_1, p_2) - \rho_1(p_1)\rho_1(p_2)$$ (1)
where \( \rho_2(p_1,p_2) = \frac{dN}{dy_1dy_2dp_1dp_2} \) is the density of particle pairs and \( \rho_1(p) = \frac{dN}{dydp} \) is the single particle density.

While both jets and hydrodynamic transverse flow give increased transverse momentum to particles, jets produce a greater number of particles \( N \) and a greater amount of \( p_t \) per particle. If \( p_t \) is uncorrelated with multiplicity \( N \), we can write

\[
(NP_t) - (N)(N_t) = \langle p_t \rangle (\langle N^2 \rangle - \langle N \rangle^2)
\]  

(2)

We expect jets to increase the covariance of \( p_t \) and \( N \) making the left side larger than the right side of Eq. (2). On the other hand, flow does not affect \( N \) but does increase the \( p_t \) of particles as they are pushed away from the beam. This effect increases the right side of Eq. (2) due to the increased \( \langle p_t \rangle \), but leaves the left side unchanged.

These observations lead us to propose that the quantity

\[
D = \frac{1}{(N)^2} \left[ (\langle NP_t \rangle - (N)(N_t)) - \langle p_t \rangle \langle \langle N^2 \rangle - \langle N \rangle^2 \rangle \right]
\]  

(3)

will be positive in the presence of jets and negative in events where flow is present. This gives us a calculation to determine between jets and flow.

We relate \( \mathcal{D} \) to the pair density function above as follows

\[
\langle N \rangle^2 \mathcal{D} = \int dp_1dp_2(r(p_1,p_2))\langle p_1 \rangle \langle p_2 \rangle
\]  

(4)

and will now show this form is equivalent to that in Eq. (3). I continue to closely follow Catanzaro’s work in defining \( \langle N \rangle = \int \rho_1 dp \) and \( \langle p_t \rangle = \langle p_t \rangle / \langle N \rangle \). These quantities can also be written as the average of the sum of all particles produced in a collision e.g. \( \langle p_t \rangle = \langle \sum p_t \rangle \) where \( p_t \) is the transverse momentum of the \( i \)-th produced particle. It is also true that the integral of the pair density satisfies \( \int \rho_2 dp_1dp_2 = \langle N - 1 \rangle \).

Remembering the two-body correlation function \( r \) (Eq. (1)), I now expand \( \mathcal{D} \) from Eq. (4) as

\[
\langle N \rangle^2 \mathcal{D} = \int dp_1dp_2(\langle p_1 \rangle - \langle p_t \rangle)[\rho_2(p_1,p_2) - \rho_1(p_1)\rho_1(p_2)]
\]  

(5)

The first term can be written as

\[
\int p_1\rho_2(p_1,p_2)dp_1dp_2 = \langle N - 1 \rangle p_t
\]

\[
= \langle NP_t \rangle - \langle p_t \rangle
\]

\[
= \langle NP_t \rangle - \langle p_t \rangle(N)
\]  

(6)

The second term, \( \int p_1\rho_1dp_1dp_2 = \int dp_1\rho_1(p) \int dp_2(p) \) is \( \langle p_t \rangle \langle N \rangle \) by definition. Finally the last two terms combine in the following way

\[
\int dp_1dp_2[-\langle p_t \rangle \rho_2 + \langle p_t \rangle \rho_1] = \int dp_1dp_2[-\langle p_t \rangle (\rho_2 + \rho_1)]
\]

\[
= \int dp_1dp_2[-\langle p_t \rangle r]
\]
where \( R \) is defined as

\[
R = \frac{1}{(N)^2} \int dp_1 dp_2 r(p_1, p_2) = \frac{(N^2) - \langle N \rangle^2 - \langle N \rangle}{(N)^2}
\]  

(8)
as is discussed in [2].

These parts can now be combined and we find

\[
\langle N \rangle^2 D = \int dp_1 dp_2 (p_{1t} - \langle p_{1t} \rangle)r(p_1, p_2)
\]

\[
= \langle NP_t \rangle - \langle N \rangle \langle P_t \rangle - \langle p_{1t} \rangle \langle N \rangle - \langle p_{1t} \rangle (\langle N(N - 1) \rangle - \langle N \rangle^2)
\]

\[
\langle N \rangle^2 D = \langle NP_t \rangle - \langle N \rangle \langle P_t \rangle - \langle p_{1t} \rangle (\langle N^2 \rangle - \langle N \rangle^2)
\]

This is the same form as that given in Eq. (3) and is the one I used for calculations while running HIJING.

3 Independent Source Model

The ability to scale the values calculated for \( D \) in p-p events to that in Au-Au events relies heavily on the idea of the independent source model. This describes nuclear collisions as a superposition of hadron collisions and neglects any re-scattering effects. It implies that charged-particle pairs are only correlated if they are created in the same sub-collision [2].

If we say there are \( M \) sources, we can write \( p_{1t} = \bar{\rho}_1 M \). We will identify the number of sources \( M \) with the number of participants, as is appropriate for comparison to HIJING. However, we stress that the derivation from (9) to (10) depends only on the assumption that the sources are independent from one another. We can then find a relationship between \( D \) for p-p events and for Au-Au events as follows. We first see \( \langle N \rangle \) can be written as

\[
\langle \int dp p_{1t}(p) \rangle = \langle \int dp \bar{\rho}_1(p) \rangle M = \langle \int dp \bar{\rho}_1(p) \rangle (M)
\]

(9)

We define \( \mu = \int dp \bar{\rho}_1(p) \) giving \( \langle N \rangle = \mu M \) and \( \rho_2 = \bar{\rho}_2 M + \bar{\rho}_1 \bar{\rho}_1 M (M - 1) \). Starting with its definition, we see \( D \) can now be written as

\[
D = \frac{1}{(N)^2} \int dp_1 dp_2 (p_{1t} - \langle p_{1t} \rangle)r(p_1, p_2)
\]

\[
= \frac{\int dp_1 dp_2 (p_{1t} - \langle p_{1t} \rangle)r(p_1, p_2)}{(\int p_{1t})^2}
\]

\[
D = \frac{b_0 - \mu^2 p_{1t}}{\mu^2(M)} + p_{1t} \left[ \frac{(M^2) - \langle M \rangle^2}{(M^2)} \right] - p_{1t} \left[ \frac{a_0 - \mu^2}{\mu^2(M)} + \frac{(M^2) - \langle M \rangle^2}{(M^2)} \right]
\]

where \( b_0 = \int \bar{\rho}_2 p_{1t} \) and \( a_0 = \int d^3 p_1 d^3 p_2 p_{1t} \). Continuing on we see this can be simplified as

\[
D = \frac{b_0 - \mu^2 p_{1t}}{\mu^2(M)} - \frac{p_{1t}(a_0 - \mu^2)}{\mu^2(M)}
\]

(10)

We see that the contribution from fluctuations in the number of sources cancels out. We finally obtain:

\[
D = \frac{b_0 - \mu^2 p_{1t}}{\mu^2(M)}
\]
In a nuclear collision, the number of sources is the same as the number of participant nucleons in the event, \( N_{\text{part}} \). The number of participants in a p-p event is two. Using Eq. (10) we find

\[
D_{AA} = \frac{2D_{pp}}{N_{\text{part}}} \tag{11}
\]

which relates the value for \( D \) in p-p events to that found in nuclear events. Similarly, it has been shown in Ref. [2] that

\[
R = \frac{2R_{pp}}{N_{\text{part}}} + \frac{\text{Var}(N_{\text{part}})}{N_{\text{part}}^2}, \tag{12}
\]

where \( \text{Var}(N_{\text{part}}) \) is the variance of the number of participants; see e.g., Catanzaro’s report. We stress that (11) and (12) depend on the independence of the participant sources.

### 3.1 HIJING

As stated above, HIJING is a Monte Carlo event generator designed for the study of high energy pp, pA, and AA collisions [1]. HIJING uses subroutines from a simulator of elementary particle collisions, PYTHIA to create a model for nuclear collisions. Of interest to us is the fact that HIJING is a superposition of these PYTHIA events. This model is useful as we test the formulas above based on the independent source model.

### 4 Results

In this section I explore the effect of jet quenching on fluctuations and correlations in Au-Au collisions using a sample of 16,000 Au-Au collisions. This sample is adequate to illustrate the physics, but it is too small to provide concrete estimates. The results I did obtain are therefore preliminary.

To begin, I simulated p-p events and compared them with Catanzaro’s PYTHIA results and data from [3]. For these events, in the pseudorapidity range of \( |\eta| \leq 1 \), I found \( D_{pp} = 0.0278 \) while Catanzaro calculated \( D_{pp} = 0.0235 \). Scaling these values using Eq. (11), I calculate \( D_{AA,\text{scaled}} = 1.455 \times 10^{-4} \) with Catanzaro obtaining \( D_{AA,\text{scaled}} = 1.230 \times 10^{-4} \).

I was then able to run just over 16,000 Au-Au events. I calculated \( D \) for these events in the same range of pseudorapidity with HIJING’s switch for jet quenching turned off. Thinking there may have been interesting physics related to \( D \) with jet quenching, I ran more Au-Au events with this switch turned to on. The results of the simulations can be seen in Table 1. In this instance, I found about a factor of ten difference between the calculated \( D \) and the value from the scaling relation.

Table 2 shows that jet quenching reduces the magnitude of \( D \) while increasing the magnitude of \( R \). To understand these trends, we first focus on \( R \). Observe that the values for \( R \) in AA collisions agree with the scaling from pp in sec. 3, but only when jet quenching is turned off. We understand this as follows: In the absence of jet quenching, HIJING produces particles from nucleon collisions that are essentially independent but for the global conservation of energy and momentum. Hence an independent source model should work reasonably well with quenching off. Quenching results from the scattering of jet particles with other particles produced in the event, i.e., the medium. The degree to which a particular jet is quenched depends on its path through the medium as well as the density and motion of the medium. This medium dependence adds correlations and, correspondingly, the quenched simulations deviate from independent-source scaling.
Table 1- Calculated values for average number of participants, number of events, \(<N>\) for charged particles, \(<p_\text{t}>\), \(R\), and \(D\) for PYTHIA events and HIJING events with the switch for jet quenching turned off and on within the range \(|\eta|\leq1\).

<table>
<thead>
<tr>
<th></th>
<th>PYTHIA</th>
<th>HIJING (no jet-quenching)</th>
<th>HIJING (jet-quenching)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Participants</td>
<td>2</td>
<td>381.57782</td>
<td>381.60403</td>
</tr>
<tr>
<td>Number of Events</td>
<td>5000000</td>
<td>16100</td>
<td>16100</td>
</tr>
<tr>
<td>(&lt;N&gt;)</td>
<td>5.1917</td>
<td>1619.0014</td>
<td>1948.146</td>
</tr>
<tr>
<td>(&lt;p_\text{t}&gt;)</td>
<td>0.4105</td>
<td>0.4512</td>
<td>0.4269</td>
</tr>
<tr>
<td>(R)</td>
<td>0.5853</td>
<td>0.0033</td>
<td>0.0042</td>
</tr>
<tr>
<td>(D)</td>
<td>0.0278</td>
<td>7.780\times10^{-5}</td>
<td>3.698\times10^{-5}</td>
</tr>
</tbody>
</table>

Table 2- Comparison between values calculated using the scaling relations for \(D\) and \(R\) and those found with HIJING’s switch for jet quenching turned off and on.

<table>
<thead>
<tr>
<th></th>
<th>Scaled from p-p</th>
<th>No Jet Quenching</th>
<th>Jet Quenching</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D)</td>
<td>1.455 \times10^{-4}</td>
<td>7.780 \times10^{-5}</td>
<td>3.698 \times10^{-5}</td>
</tr>
<tr>
<td>(R)</td>
<td>0.0032</td>
<td>0.0033</td>
<td>0.0042</td>
</tr>
</tbody>
</table>

To see why quenching increases \(R\) and understand its effect on \(D\), we must understand how quenching satisfies relativistic energy and momentum conservation. Quenching reduces the jet momentum both by adding momentum to the low \(p_\text{t}\) “soft” particles (elastic scattering) and by adding additional soft particles (inelastic scattering). In string fragmentation models like HIJING, the inelastic production of soft particles is the dominant process. This is also the case in many QCD-based jet quenching models. Therefore the increase \(R\) in Table 2 follows from the added fluctuations of multiplicity introduced by adding soft particles.

This explanation implies that jet quenching should reduce the value of \(D\) and, indeed, this is supported by the results in Table 2. One remaining issue is that \(D_{AA}\) in my simulations does not seem to follow the independent-source scaling in sec. 3. This could be contributed to the lack of statistics. \(D\) is the difference of two large terms of comparable magnitude, \((Np_t) - \langle N\rangle\langle p_t\rangle\) and \((\langle N^2\rangle - \langle N\rangle^2\). My exploratory 16,000 event sample is too small to compute this difference. Work is in progress.

5 Summary

I have been working alongside Sean Gavin and Michael Catanzaro studying momentum-multiplicity correlations in high-energy collisions using a derived quantity known as \(D\). I showed the derivation of a scaling relation between the calculated value for \(D\) in p-p events to that in nuclear collisions. I was unable to give a firm conclusion to the validity of the relationship due to a lack of statistics. Further study is needed to show this relationship.

6 Further Work

The calculation of \(D\) is still in progress. Currently Catanzaro is finishing his analysis of \(D\) in PYTHIA studying the effect of jets on the calculation. G. Moschelli along with Catanzaro has done work related to the blast-wave model and may be conducting further studies using the Monte Carlo simulation package URQMD [4] which can model hydrodynamic flow. These studies can then be combined to discover the roles jets and flow have in momentum correlations.
Acknowledgements

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References