Search for Correlations in Supernova Light Curves Beyond the Standard Two-Parameter Model

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Abstract

Using supernovae from the SDSS-SN Survey we fit the shape parameter to individual SDSS passbands (g, r, i) as well as simultaneously to all passbands using SALT2. Through checking the normalized residuals, the passbands seem consistent when compared band to band. The i passband appears to deviate from the shape as measured in all passbands. There are hints of an effect for shape versus color in the r and i passbands that are possibly interesting. For the i passband, an adjustment in the i passband SALT2 model shape would need to occur. The correlations in the color parameter are not currently built into the SALT2 model, and would require a revised SN light curve model. However, the error calculations for slope calculations were oversimplified and the statistical significance of the effect is in doubt.

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1. Introduction

A type I supernova occurs when a white dwarf gains too much mass triggering a thermonuclear reaction causing intense brightness. This brightness is roughly the same in all supernovae and can outshine entire galaxies. Since the absolute brightness is well known, supernovae apparent brightness are used to measure distances and consequently the expansion of the universe. The rate of expansion of the universe gives us constraints on the properties of dark energy.

This project studied confirmed type Ia supernova found in the Sloan Digital Sky Survey (SDSS)-II Supernova Survey (Frieman et al. 2008). Type Ia supernovae are used in part because of their ease of identification. The spectra of type Ia supernovae contain no hydrogen nor do they contain helium. However, they do contain silicon making their spectra unique. Type Ia supernovae are also known as standard candles as their absolute magnitude is well known. Our data was from the three campaigns of the SDSS-II SN Survey; 2005, 2006, and 2007.

The SDSS-II SN Survey (Frieman et al. 2008) has discovered over 500 spectroscopically confirmed supernovae. Data was collected at the Apache Point Observatory, New Mexico using a 2.5 meter telescope (Gunn et al. 2006). This telescope uses the SDSS CCD camera which acquires images in the five optical passbands, ugriz (Gunn et al. 1998). These optical passbands range from 3000 Å to 11000 Å and are divided into five nearly passbands (Fukugita et al. 1996) as shown in Figure 1.

Fig. 1. – The above plot shows the quantum efficiency as a function of wavelength (Å) for each SDSS passband.

The CCD camera uses a technique called time-delay-and-integrate (TDI) drift scan to efficiently explore the sky (Gunn et al. 2006). Repeat imaging is used to detect supernovae. By subtracting images, transient events are found. SN candidates are analyzed spectroscopically and photometrically and are compared to Monte Carlo simulated SN and confirmed supernovae from previous surveys. Light curve properties are compared as were spectra for identification of type Ia supernovae (Sako et al. 2008).

For comparisons, Monte Carlo simulated supernovae were used through the SNANA (Kessler et al. 2009b) simulator and light curve points were fitted using the SALT2 light curve fit parameter on both the simulated and real data using the SNANA light curve fitter. SALT2 uses measured light curve points and determines parameters (Guy et al., 2007).

The SALT2 fit parameters are shape (x_i), excess color (c), and apparent brightness (m_i). The apparent brightness is simply an overall scale factor, so the SALT2...
can be thought of as a two-parameter model. The model uses \( x_1 \) to describe the shape and light curve of the SN and \( c \) to describe the excess color (redder or bluer) of a SN relative to the nominal SN with shape parameter \( x_1 \) (Guy et al., 2007). The parameters \( x_1 \) and \( c \) are used to reduce the scatter in the Hubble diagram (Kessler et al. 2009a). The \( x_1 \) parameter describes the shape of the light curve in each band. The shape in each band is well-described by the single parameter \( x_1 \) which distinguishes broader light curves from narrower ones. The length of a light curve depends on the energy of explosion, or the energy from the nuclear reaction in a supernova, which gives us the light output. In SALT2, \( x_1 \) is measured using a combination of all SDSS passbands.

The color of a supernova may be influenced by variations in the explosion process. Since energy is proportional to frequency, the bluer the color, the hotter the explosion and the brighter the event. Color is the ratio of flux in one passband versus another passband. In SALT2 the \( B \) and \( V \) passbands (Johnson-Morgan-Cousins passbands) are used for to measure color:

\[
\begin{align*}
    m_B &= -2.5 \log_{10} f_B \\
    m_V &= -2.5 \log_{10} f_V \\
    c &= -2.5 \log_{10} \frac{f_B}{f_V} = m_B - m_V
\end{align*}
\]

where \( f_B \) and \( f_V \) are the measured flux in the \( B \) and \( V \) passbands, respectively, and \( m_B \) and \( m_V \) are the apparent magnitudes. The data was measured using the SDSS passbands; however, the color parameter is translated into Johnson-Morgan-Cousins passbands. The color is also affected by host galaxy dust which is difficult to distinguish from excess color due to variations in the explosion process.

This project uses SALT2 to fit \( x_1 \) for each band separately as well as the usual multi-band fit for \( x_1 \) and \( c \). The differences in \( x_1 \) for each band is compared with the combined fit \( x_1 \) and correlated with \( c \) in different passbands. The goal is to measure the size of deviations of the \( x_1 \) measured for each passband individually and see if deviations are correlated with any other quantity.

2. Data Sample: Light Curve Fit Cuts

The light-curve selection criteria for SN type Ia is based on cuts made in the SNANA light curve fitter. For these criteria, a measurement corresponds to a recorded viewing of a SN in the SDSS-II survey. Redshifts refer to the increase in the wavelength of the radiation emitted (shift in spectral lines) due to the expansion of the universe with time. \( T_{\text{rest}} \) refers to the time in days in the rest frame of the SN which is different on the earth due to relativistic effects. In order to have accurate fit parameters for SN, it is important to have good time sampling as well as good signal to noise ratio (SNR). Thus cuts are made on these parameters to give acceptable SN light curves. SN light curves must follow these criteria:

- Redshifts must be in the range of \( 0.0 < z < 0.45 \)
- One measurement is required at a \( T_{\text{rest}} < -2 \) days (two days before the SN event)
- Five measurements must be taken within the range of \( -20 < T_{\text{rest}} < +80 \) days (20 before the event to 80 days after)
- One measurement is required at a \( T_{\text{rest}} > 10 \) days (measurement 10 days after event)
- One measurement is required to have a \( \text{SNR} > 5.0 \)
- Five measurements are required to have a \( \text{SNR} > 2.0 \)

After these cuts were defined and the light curves were made, the \( x_1 \) and \( c \) were plotted for the SN that made the cuts as shown in Fig. 2.

![Fig. 2. The \( x_1 \) measured in all passbands has a bell curve distribution. The left-hand panel is data; the right-hand panel is the simulation.](image)

![Fig. 3. The \( c \) parameter as measured in all passbands. The left-hand panel is data; the right-hand panel is the simulation.](image)

3. Data Sample: Improvement Cuts

The SALT2 model expectation is that \( x_1 \) as measured in all passbands should be the same in each individual passband:

\[
x_1 = x_{1u} = x_{1g} = x_{1r} = x_{1i} = x_{1z}.
\]

To test this expectation, \( x_{1g} \) was plotted against \( x_1 \) in Fig. 4.

The data in Fig. 4. a. largely follows this expectation except that there is a band of outliers at \( x_1 \approx 5 \). This effect is best seen in the \( g \) passband plots of both real data (see
Fig. 4. a.) and simulated data and in the $i$ passband plot for simulated data. These outliers have poorly measured $x_1$ values. These supernovae were cut from the data sample as shown in Fig. 4. b., greatly reducing the problem.

The next step in analyzing the data sample was to plot the $c$ parameter against what the analysis refers to as delta $x_1$. Delta shape ($\Delta x_1$) refers to:

$$\Delta x_{1g} = x_{1g} - x_1$$

where the subscripted letter refers to the passband specified.

To test for correlations in the data, a slope line was fit to the data. For the purpose of computing errors, it was assumed that:

$$x_1 = \frac{x_{1g} + x_{1r} + x_{1i}}{3}.$$  

This is not rigorously true, but is a reasonable estimate. The errors were calculated through the following reasoning (example for the $g$ passband):

$$x_{1g} - x_1 = \frac{2}{3} x_{1g} - \frac{1}{3} x_{1r} - \frac{1}{3} x_{1i},$$

$$\sigma(x_{1g} - x_1) = \sqrt{\frac{1}{9} \sigma x_{1g}^2 + \frac{1}{9} \sigma x_{1r}^2 + \frac{1}{9} \sigma x_{1i}^2}.$$  

where $\sigma x_{1g}$ is the error on $\Delta x_{1g}$.

To see if the data set was statistically significant, a test on the number of standard deviations was done, where standard deviations were calculated using the equation:

$$\frac{s_m - s_0}{\sigma}$$

where, $s_m$ refers to the measured slope and $s_0$ refers to the slope used for comparison, in our case, $s_0$ was set to 0 to see if a slope existed. The symbol $\sigma$ refers to the slope error, which is explained above. Theoretically, there is approximately a 68.2% chance that the data sample will be within $1\sigma$, a 95.4% chance of $2\sigma$, and a 99.7% chance of the data being within $3\sigma$.

The plots in Fig. 5. were fit to a slope line to test correlations between $c$ and $x_1$ fit to the $r$ passband. These slopes and their errors were tested to see if they were significantly different from zero. The slope for the real data was 1.84 and the standard deviation for the $r$ passband slope was 8.9, which indicates that the slope is not zero. For the simulated data, the fitted slope is quite small at -0.1206. However, because of the larger number of SN in the simulation, the slope is not very consistent with zero at -2.3 standard deviations.

For the first evaluation of this plot, it was apparent that there was a relatively large slope in the $r$ passband of the real data. Upon closer evaluation of the $\Delta x_1$ values, it was clear that there was an outlier point with a $\Delta x_1$ of 7.9146, which greatly affected the slope. There were no such points in the simulation and we rejected the outlier. The light curve of the outliers while giving a good light curve fit is characterized by having no points on the rising part of the curve and a possibly large fluctuation in the magnitude of the peak and late-time $r$ passband points. The light curve is shown in Fig. 6.
After the supernova was cut out, the plots were redone to get a better measurement of a possible slope correlation between $c$ and $\Delta x_{1r}$. The slope measurement decreased as shown in Fig. 7, and the line is a much better fit to the bulk of the data.

A cut on $c$ was made due to the reddening from extinction. Color variation is caused by variation in supernova explosions as well as dust trapped in galaxies. Reddening from galaxies comes from both the host galaxy of the SN and the Milky Way galaxy. There is fairly good knowledge of what this extinction value is for the Milky Way galaxy.

Extreme values of $c$ are usually caused by significant dust in host galaxies, where as the middle range of values are caused by the variations in explosion of the SN with some modest extinction. It is expected that host galaxy dust would not be correlated with $x_1$ but it might be correlated with other color variations in the explosion process. To limit the influence of host galaxy extinction we consider only SN in the $c$ range $-0.2 < c < 0.2$. Included was a cut made for all data on the error of how well $x_1$ was measured. It is ideal for the data to have well measured $x_1$ values and to not include poorly measured light curves.

4. Results

In Fig. 8., the normalized residuals of $\Delta x_{1gr}$, $\Delta x_{1gi}$, and, $\Delta x_{1ri}$. The RMS values for these plots should be 1.0. Since all are less than one as shown in Table 1, it can be inferred that the errors for the $x_1$ values were overestimated in both the real and simulated data. The data is not broader than the Monte Carlo simulation. We expected that the normalized residuals would be accurate since they originate from a straight-forward calculation in the SNANA program. Although the error appears to be overestimated, we can conclude that there is no evidence that the $\Delta x_1$ exceeded the values expected solely from statistical fluctuations. However, we can still search for correlations that would not be expected in the SALT2 model.

Fig. 6. – Above is the cut supernova’s light curve which has an anomalous $x_1$ measurement.

Fig. 7. – The $c$ parameter is plotted against $\Delta x_{1r}$ for real data. However, the real data was cut by an outlier supernova (right plot).

Fig. 8. a. – This plot shows the normalized residuals as compared in the $g$ and $r$ passbands for real and simulated data.

Fig. 8. b. – The above plot shows the normalized residuals as compared in the $g$ and $i$ passbands for real (left) and simulated (right) data.

Fig. 8. c. – This plot shows the normalized residuals for the $r$ and $i$ passbands for real and simulated data.
The Fig. 9. shows the $x_1$ parameter plotted against the $\Delta x_1$ parameter for each passband ($g, r, i$). Slope lines were calculated for significance to determine correlations. The $i$ band shows an apparently significant slope with 4.2 standard deviations. The plots also show that the simulated data also show statistically significant slopes, however, these slopes are 5 times smaller. It is not clear what causes the simulated slopes to deviate from zero but it could be our naïve error calculation that probably is an underestimate of the true error. In what follows, we will neglect the small but statistically significant slopes seen in the simulation. The larger slopes seen in the data will be considered potentially interesting although they could suffer from the same underestimate of errors that is a possible issue with the simulated data. If the $x_1 i$ passband correlations really are true, a change in the SALT2 model for the $i$ passband would be indicated.

<table>
<thead>
<tr>
<th>Normalized Error</th>
<th>Mean</th>
<th>RMS</th>
<th>Error in Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Data</td>
<td>g - r</td>
<td>-0.1018</td>
<td>0.2251</td>
</tr>
<tr>
<td></td>
<td>g - r</td>
<td>0.09734</td>
<td>0.2267</td>
</tr>
<tr>
<td>Sim Data</td>
<td>g - i</td>
<td>-0.1822</td>
<td>0.2369</td>
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<tr>
<td></td>
<td>g - i</td>
<td>0.09436</td>
<td>0.2307</td>
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<tr>
<td>Real Data</td>
<td>r - i</td>
<td>-0.08632</td>
<td>0.1508</td>
</tr>
<tr>
<td></td>
<td>r - i</td>
<td>-0.00273</td>
<td>0.1992</td>
</tr>
</tbody>
</table>

Table 1. – This table shows the mean and RMS of the normalized residuals.

Fig. 9. a. – The $x_1$ measured in all passbands is plotted against $\Delta x_1$ for both real and simulated data. These plots include all cuts as described in § 2 and 3.

Fig. 9. b. – The $x_1$ measured in all passbands is plotted against $\Delta x_1$, for both real and simulated data. Again, these plots include all cuts as described in § 2 and 3.

Fig. 9. c. – The $x_1$ measured in all passbands is plotted against $\Delta x_1$, for both real and simulated data with all cuts included.

<table>
<thead>
<tr>
<th>Plot</th>
<th>Slope</th>
<th>Slope Error</th>
<th>Standard Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Data</td>
<td>x1 vs $\Delta x_1$ (g)</td>
<td>0.0370</td>
<td>0.0278</td>
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<tr>
<td>Sim Data</td>
<td>x1 vs $\Delta x_1$ (g)</td>
<td>-0.0473</td>
<td>0.0076</td>
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<tr>
<td>Real Data</td>
<td>x1 vs $\Delta x_1$ (r)</td>
<td>0.0293</td>
<td>0.0279</td>
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<tr>
<td>Sim Data</td>
<td>x1 vs $\Delta x_1$ (r)</td>
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<td>0.0064</td>
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<tr>
<td>Real Data</td>
<td>x1 vs $\Delta x_1$ (i)</td>
<td>0.1307</td>
<td>0.0314</td>
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<tr>
<td>Sim Data</td>
<td>x1 vs $\Delta x_1$ (i)</td>
<td>-0.0223</td>
<td>0.0069</td>
</tr>
</tbody>
</table>

Table 2. – This table shows the slope and standard deviations of the $x_1$ vs. $\Delta x_1$ plots.

Fig. 10. shows the $c$ parameter plotted against the $\Delta x_1$ parameter for each passband ($g, r, i$). A slope line is calculated for significance to search for correlations. The plots in Fig. 10. b. and 10. c. show a slightly more significant slope. Fig. 10. b. has a slope of 0.9880 and 3.2 standard deviations and Fig. 10. c. has a slope of 0.8657 and 2.5 standard deviations. The standard deviations in this data set are smaller than the $x_1$ plots. Again, like the previous plots for $x_1$, the simulated data slopes appear to be significant, however, the slope measurements are much smaller than those for the real data, and thus they are ignored. A possible correlation between $\Delta x_1$ and the $c$ parameter is more fundamental and outside of the two-parameter model. If the correlations in the $c$ parameter in the $r$ and $i$ passbands hold true, there would need to be a change in the two-parameter model.
Fig. 10. a. – The $c$ parameter is plotted against $\Delta x_{1g}$ passband for both real and simulated data with all cuts included.

Fig. 10. b. – The $c$ parameter is again plotted against $\Delta x_{1r}$ for both real and simulated data.

Fig. 10. c. – The $c$ parameter is plotted against $\Delta x_{1i}$ passband for both real and simulated data with all cuts included.

### Table 3.

<table>
<thead>
<tr>
<th>Plot</th>
<th>Slope</th>
<th>Slope Error</th>
<th>Standard Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Data c vs $\Delta x_1$ (g)</td>
<td>-0.5607</td>
<td>0.3024</td>
<td>-1.9</td>
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<tr>
<td>Sim Data c vs $\Delta x_1$ (g)</td>
<td>-0.2251</td>
<td>0.0927</td>
<td>-2.4</td>
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<tr>
<td>Real Data c vs $\Delta x_1$ (r)</td>
<td>0.988</td>
<td>0.3117</td>
<td>3.2</td>
</tr>
<tr>
<td>Sim Data c vs $\Delta x_1$ (r)</td>
<td>-0.1211</td>
<td>0.0774</td>
<td>-1.6</td>
</tr>
<tr>
<td>Real Data c vs $\Delta x_1$ (i)</td>
<td>0.8657</td>
<td>0.3446</td>
<td>2.5</td>
</tr>
<tr>
<td>Sim Data c vs $\Delta x_1$ (i)</td>
<td>-0.0214</td>
<td>0.0789</td>
<td>-0.3</td>
</tr>
</tbody>
</table>

Table 3. - This table shows the slope and standard deviations of the $c$ vs. $\Delta x_1$ plots.

### 5. Conclusions

The slope calculations were made to see if there were differences in $x_1$ that got bigger or smaller depending on the $c$ of supernovae. In the data analysis, there were slopes found in the $x_1$ versus $\Delta x_{1i}$, and $c$ versus $\Delta x_{1r}$ and $\Delta x_{1i}$. These correlations were not expected nor are they built into the current two-parameter light curve model. These correlations may give a relationship to different passbands having different $c$ which would be a new feature of a revised supernova light curve model.

However, it should be noted that the error calculations for slope calculations were oversimplified approximations. For these error calculations, the horizontal axis error is ignored. Thus, the number of standard deviations may not be exact. This explains leniency with evaluating standard deviations in regards to the significance of slopes.

If slopes are really non-zero, the $x_1$ $i$ passband (see Fig. 9. c) correlations would indicate a need to adjust the $x_1$ in the $i$ passband, though this effect is not necessarily a violation of the model. However, the $c$ parameter is more fundamental and outside the model. If the correlations in the $c$ parameter in the $r$ and $i$ (see Fig. 10. b and 10. c) passbands are true, a change in the two-parameter model would need to occur.

### Acknowledgements

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### References

Gunn, J. E. et al. 1998 AJ, 116, 3040